Discussion 1 Worksheet

Curves Defined by Parametric Equations

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MATH 53 Multivariable Calculus

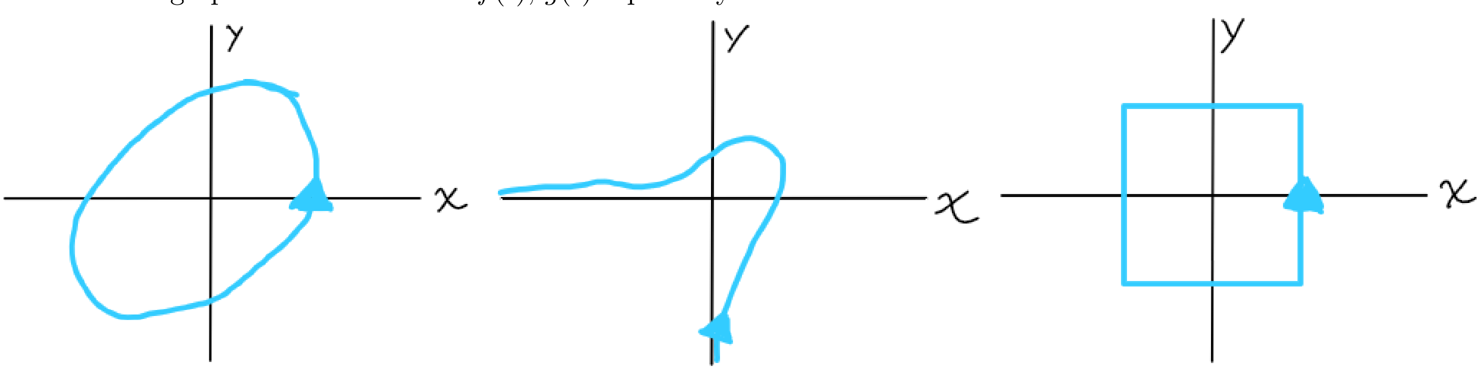
1 Recognizing Common Curves

The following table has verbal descriptions of curves in the left column, parametric equations in the center column, and verbal descriptions in the right column. Match the verbal descriptions to the corresponding Cartesian equations and parametric equations.

A circle of radius 1 centered at the origin	$x = 3\cos t,\ y = 2\sin t$	$y = 3x + 2$
A circle of radius 3 centered at $(2, 0)$	$x = t,\ y = 1/t$	$xy = 1$
A line of slope 3 passing through $(0, 2)$	$x = \cos t,\ y = \sin t$	$\frac{1}{4}x^2 + \frac{1}{9}y^2 = 1$
A hyperbola passing through $(1, 1)$ with the x and y axes as asymptotes	$x = t,\ y = 3t + 2$	$(x - 2)^2 + y^2 = 9$
An ellipse with semimajor axis of length 3 and semiminor axis of length 2	$x = 2 + 3\cos t,\ y = 3\sin t$	$x^2 + y^2 = 1$

2 Coordinate Graphs

Each of the curves below can be described parametrically by $x = f(t)$, $y = g(t)$ where f and g depend on the curve. The arrow in each graph below is positioned at the (x, y) value for $t = 0$. Draw graphs of each function $f(t)$, $g(t)$ separately.



3 Eliminating the Parameter

Find Cartesian equations $f(x, y) = 0$ for the following parametrized curves.

1. $x = \sqrt{t+1}$, $y = \frac{1}{t+1}$, for $t > -1$.

2. $x = 4 - 2t$, $y = 3 + 6t - 4t^2$.

3. $x = 2e^t$, $y = \cos(1 + e^{3t})$.

4 Sketching Curves

Sketch the following curves for $-\infty < t < \infty$ without using graphing calculators or similar tools.

1. $x = t^2 - 1$, $y = t^3 - t$
2. $x = \cos t + \frac{1}{2}\sin t$, $y = \cos t - \frac{1}{2}\sin t$
3. $x = e^t \cos t$, $y = e^t \sin t$

5 Intersections

Without using a graphing calculator (or similar), find all points (x, y) for which the following curves intersect.

1. The curve $x^2 - y^2 = 1$ and the curve $x = e^t - 2$, $y = e^t$
2. The curve $x^3 + y^2 + 1 = 0$ and the curve $x = t^4$, $y = t$
3. The curve $x = \cos t$, $y = \sin t$ and the curve $y = x^2$

6 Challenge

1. A *helix* is a curve shaped like a corkscrew. Parametrize a helix in $\mathbb{R}^3$ which goes through the points $(0, 0, 1)$ and $(1, 0, 1)$.

7 True/False

Supply convincing reasoning for your answer.

(a) T F If a curve is defined by the Cartesian equation $f(x, y) = 0$, then there are no other Cartesian equations that can be used to define that curve.

(b) T F The equations $x = 2t^3$, $y = 3t^3$ give a parametrization of a line.

Note: These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.